S 1

Sets and number sets

A set is a collection of any kind of objects (animals, people, plants, ideas, numbers ….).

A set is well defined when it is clear whether an object belongs or does not belong to the set.

A member of a set is an element of it.

Capital Latin letters will denote the sets, like A,B,C,…..,X,Y

Not capital Latin letters denote the elements of a set, like a,b,c,….,x,y

$a \in A$ — a is element of set A

$b \notin A$ — b is not element of set A

$\emptyset$ — standard notation for the empty set.

Elements of a set are collected by braces.

$$A = \{1, 2, 3, 4\}$$

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

set of the natural numbers

For many sets it is impossible or inconvenient to list the elements, instead we try to characterize the members by words or by mathemathical formulas.

$$T = \{x \mid T(x)\}$$

Example

$$B = \{n \mid n \in \mathbb{N}, n > 5\} =$$

$$= \{n \in \mathbb{N} \mid n > 5\} =$$

$$= \{6, 7, 8, \ldots\}$$

D (definition) $A = B$ when they have the same elements.

all $= \forall$

$\forall a \in A \Longrightarrow a \in B$ and

$\forall b \in B \Longrightarrow b \in A$

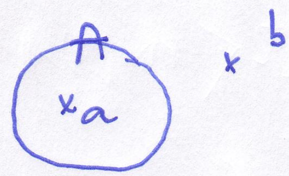
D For a set A containing only elements of B (but not necessarily all members) we write

$A \subset B$

(A is a subset of B, A is contained in B)

$(A \subset B \Longleftrightarrow \forall a \in A \Rightarrow a \in B$

Venn diagram



$a \in A$
$b \notin B$

$A \subset B$

T (theorem) $A \subset A$

T $A = B \Longleftrightarrow A \subset B$ and $B \subset A$

D A is a proper subset of B if

$A \subset B$ And $A \neq B$

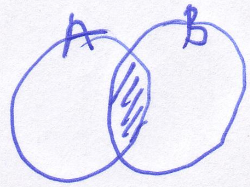
Set operations

D Union of sets

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

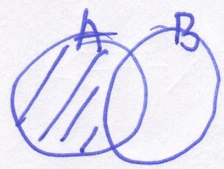
D Intersection of sets

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



D Difference of sets

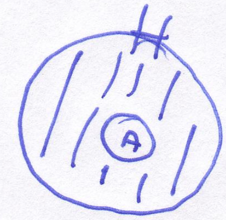
$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$



D Complementary set

$$A \subset H$$

$$\bar{A} = \{x \mid x \notin A \text{ and } x \in H\} =$$



$$= H \setminus A$$

A's complementary set H is $\bar{A}$

Set of numbers

Natural numbers

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

Integers

$$\mathbb{Z} = \{\pm n \mid n \in \mathbb{N}\} \cup \{0\}$$

Rational numbers

$$\mathbb{Q} = \left\{ \frac{m}{n} \mid m \in \mathbb{Z},\ n \in \mathbb{N} \right\}$$

A rational number can be expressed as finite or infinite decimal fraction with regular repetition of digits.

Irrational numbers

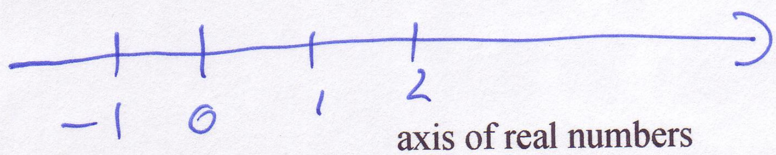
are the infinite decimal fractions without regular repetition of digits.

$$\mathbb{Q}^*$$

Real numbers

$$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^*$$

The real number line



axis of real numbers

T

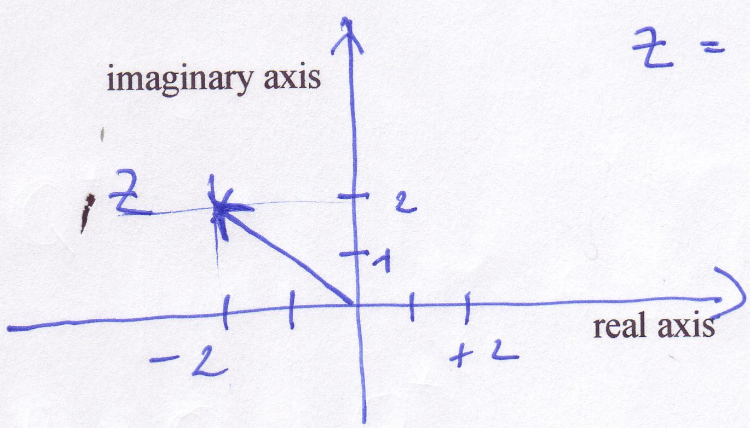
$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Complex numbers

$$\mathbb{C} = \{ a + bi \mid a, b \in \mathbb{R},\ i = \sqrt{-1} \}$$

The complex number plane

$$z = -2 + 2i$$



Operations with complex numbers

$$z_1 = a + bi \quad , \quad z_2 = c + di$$

Addition

$$z_1 + z_2 = a + bi + c + di =$$

$$= (a + c) + i(b + d)$$

Multiplication $z_1 \cdot z_2 = (a+bi)(c+di) =$

$= ac + adi + cbi + bdi^2 =$

$= (ac - bd) + i(ad + cb)$

$i^2 = -1$

Division $k \in \mathbb{R}$

$\frac{z_1}{k} = \frac{a+bi}{k} = \frac{a}{k} + \frac{b}{k} i$

Complex conjugate of a $z \in \mathbb{C}$ is z^*

$z = a + bi \qquad z^* = a - bi$

$z \cdot z^* = (a+bi)(a-bi) = a^2 + b^2$

$$\frac{z_1}{z_2} = \frac{a+bi}{c+di} =$$

$$= \frac{(a+bi)(c-di)}{(c+di)(c-di)} =$$

$$= \frac{ac - adi + bci - bdi^2}{c^2+d^2} =$$

$$= \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}$$

$\mathbb{R} \subset \mathbb{C}$